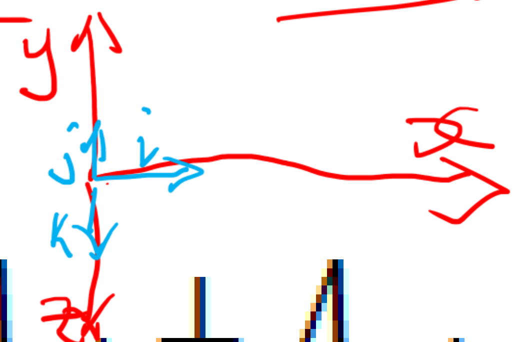
Component form of vectors

- A vector A can be written as the sum of three vectors each parallel to a coordinate



$$A = A_x i + A_y j + A_z k$$

ADDITION OF VECTORS

$$A = A_{xi} + A_{yj} + A_{zk}$$

$$B = B_{xi} + B_{yj} + B_{zk}$$

$$A + B = \left(A_{xi} + B_{xi}\right) + \left(A_{yj} + B_{yj}\right) + \left(A_{zk} + B_{zk}\right)$$

Question

What is the sum in unit vector notation of the vectors A and B?

$$A = 3i + 2j - 6k$$

$$B = -9i - 5j + 10k$$

$A + B = (3 + -9)i + (2 + -5)j + (-6 + 10)k$

$A + B = -6i - 3j + 4k$

Solution

$$A+B=\left(3i+(-9i)\right)+\left(2j+(-5j)\right)+(-6k+10k)$$

$$=-6i-3j+4k$$

MULTIPLICATION OF VECTORS

- DOT PRODUCT
- Representing a vector as a unit vector notation

$$a \cdot b = |a||b| \cos\theta \quad \text{—(i)}$$

$$a \cdot b = a_x b_x + a_y b_y + a_z b_z \quad \text{—(ii)}$$

$$a = a_x + a_y + a_z$$

$$b = b_x + b_y + b_z$$

$$a = a_{xi} + a_{yj} + a_{zk}$$

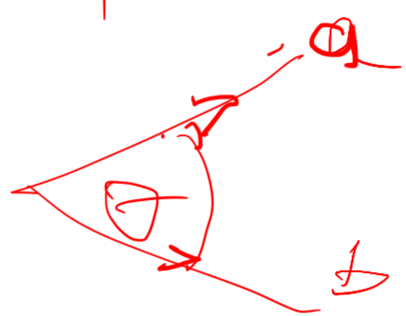
$$b = b_{xi} + b_{yj} + b_{zk}$$

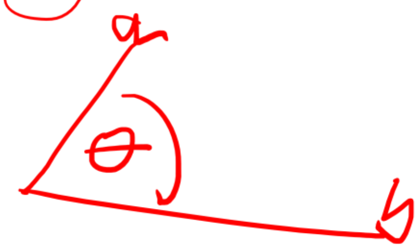
The dot product of vectors a and b is given by

$a.b = |a| \times |b| \cos\theta$

$$a.b = |a| * |b| * \cos\theta$$

$a.b = |a||b| \cos\theta$





$|a|$ is the magnitude of vector a and $|b|$ is the magnitude of vector b

$|a|$ magnitude vector a

$|b|$ magnitude vector b

$$|a|^2 = a_x^2 + a_y^2 + a_z^2$$

$$|b|^2 = b_x^2 + b_y^2 + b_z^2$$

- θ is the angle between a and b

$$|a| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$$

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$|b| = \sqrt{{b_x}^2 + {b_y}^2 + {b_z}^2}$$

$$|b| = \sqrt{b_x^2 + b_y^2 + b_z^2}$$

$a \cdot b = b \cdot a$

$4 \times 5 = 5 \times 4$

- **The dot product is commutative**

a.b=b.a

$a \cdot b = b \cdot a$

$a \cdot b = |a||b| \cos\theta$

$b \cdot a = |b||a| \cos\theta$

$3 \times 4 = 12$

$4 \times 3 = 12$

Component formula to solve a dot product of vectors in three dimensions

1 $a \cdot b = |a| \times |b| \cos\theta$

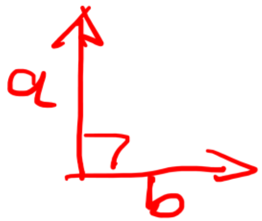
2 $$a.b = a_x b_x + a_y b_y + a_z b_z$$

$a = a_x i + a_y j + a_z k$

$b = b_x i + b_y j + b_z k$

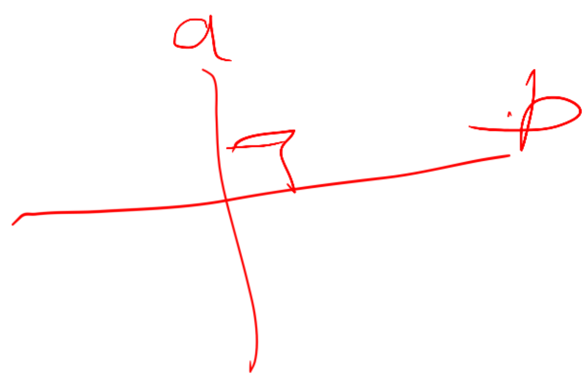
$a.b = a_x \times b_x + a_y \times b_y + a_z \times b_z$

Multiplication of perpendicular vectors



$$a.b = a_x b_x + a_y b_y + a_z b_z = 0$$

$$a \cdot b = a_x b_x + a_y b_y + a_z b_z = 0$$



Question

1. Calculate the dot product of vector a= 6i+2j+10k and b=12i+3j+5k

$* \quad a \cdot b = a_x b_x + a_y b_y + a_z b_z$

$= 6 \times 12 + 2 \times 3 + 10 \times 5$

$= 72 + 6 + 50$

$a \cdot b = 128$

Question

- Calculate the angle between the vectors a= 6i+2j+10k and b=12i+3j+5k

$a \cdot b = 6 \times 12 + 2 \times 3 + 10 \times 5 = 128$

$a \cdot b = |a||b| \cos\theta$

$128 = |a||b| \cos\theta$

$|a| = \sqrt{6^2 + 2^2 + 10^2} = \sqrt{140}$

$|b| = \sqrt{12^2 + 3^2 + 5^2} = \sqrt{178}$

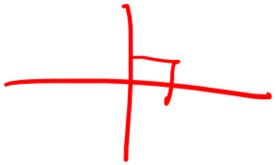
$128 = \sqrt{140} \times \sqrt{178} \cos\theta$

$\cos\theta = \dfrac{128}{\sqrt{140} \times \sqrt{178}} \Rightarrow \theta = \cos^{-1} \dfrac{128}{\sqrt{140}\sqrt{178}}$

$\underline{\theta = 35.8^\circ}$

Perpendicular vectors

- If vector a=2i+4j+2k and vector 6i+2j-dk are perpendicular, calculate the value of d.



$a \cdot b = 0$

$a_x b_x + a_y b_y + a_z b_z = 0$

$2 \times 6 + 4 \times 2 + 2 \times -d = 0$

$12 + 8 - 2d = 0$

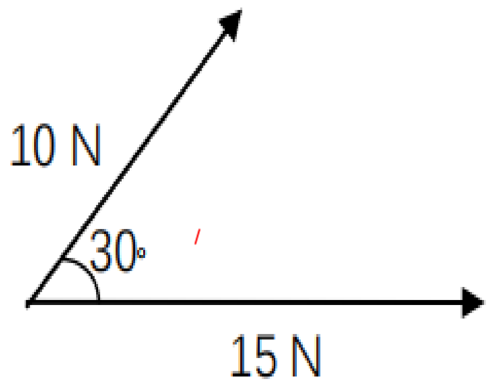
$20 - 2d = 0$

$d = 10$

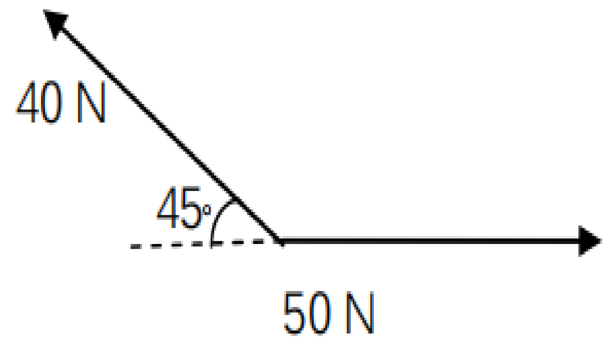
1. Two forces pull a body in different directions, 8.0 N force acts along the negative y-axis and 5.0 N force acts at 30° above the positive x-axis. Find the magnitude and direction of the resultant force.

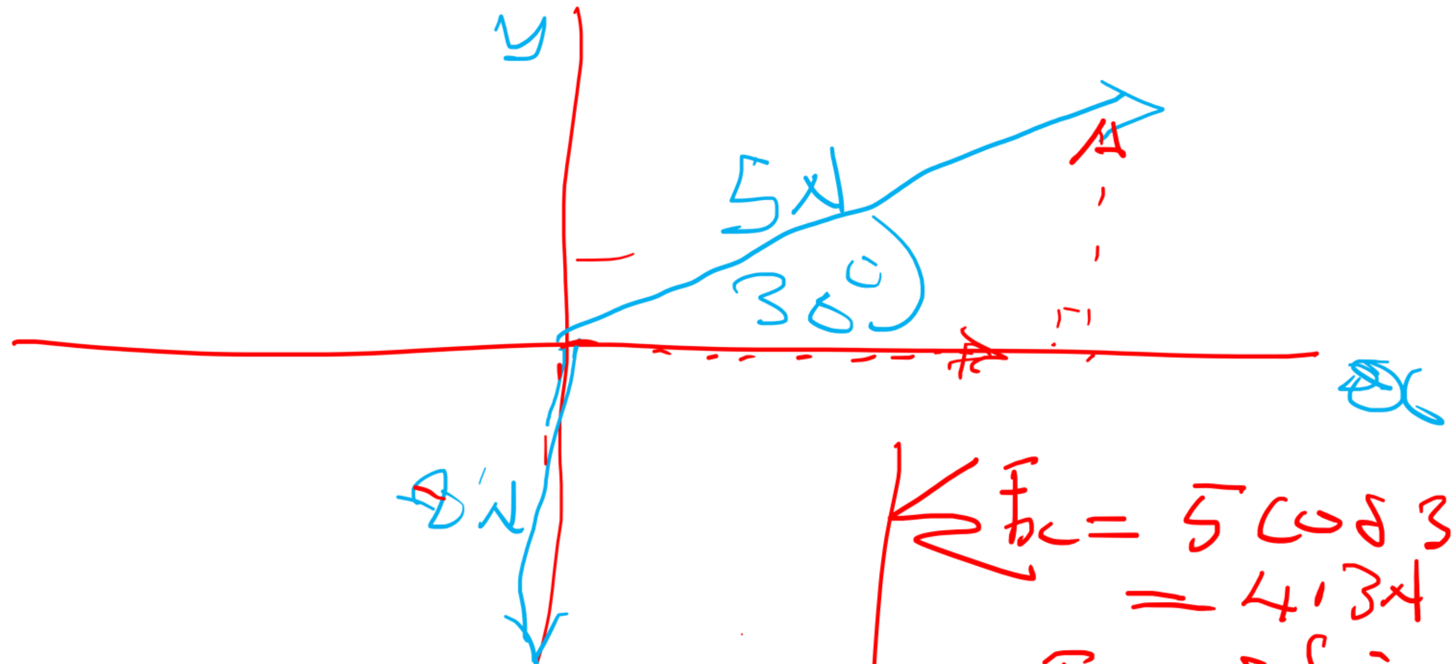
2. Find the resultant of the forces in (a) and (b)

(a)



(b)



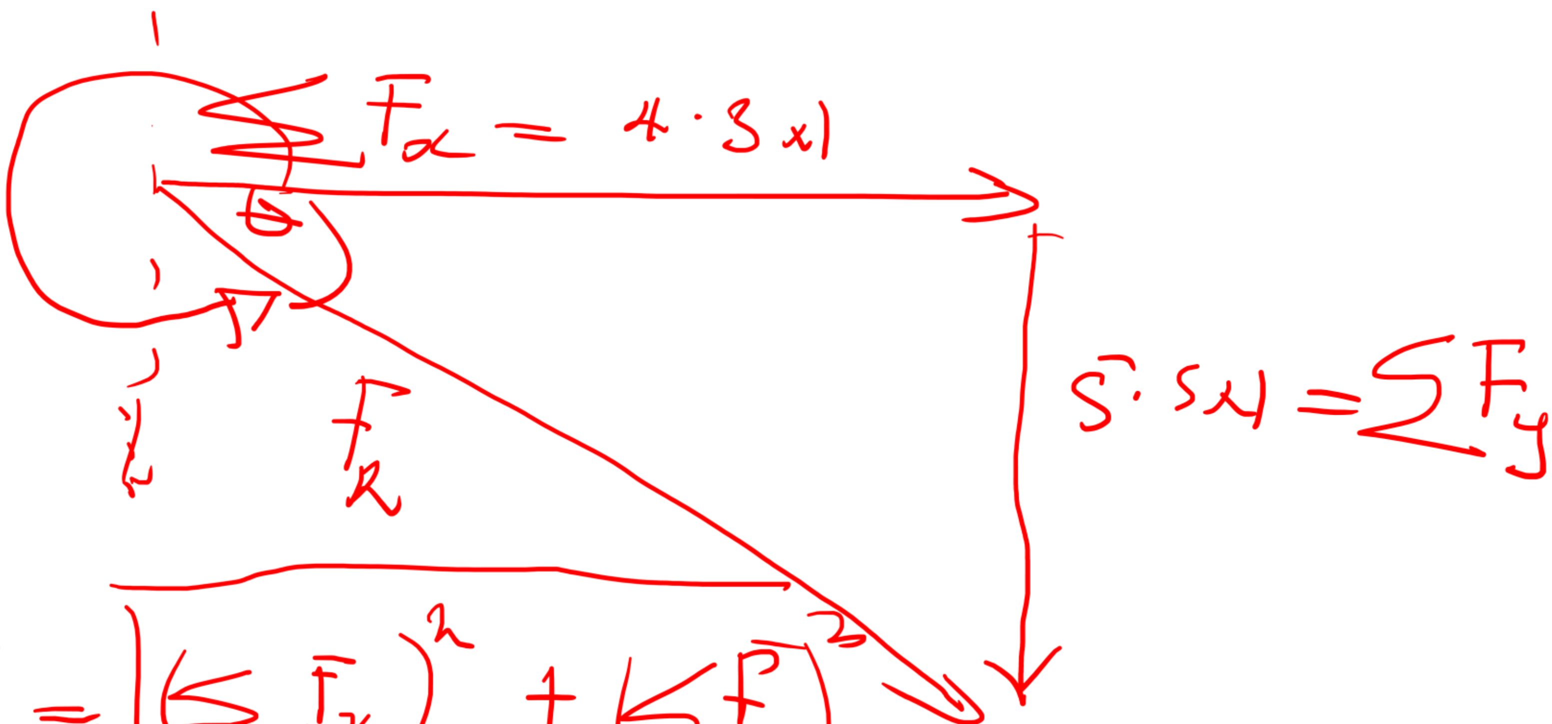


step 1

$\sum F_x$

$\sum F_y$

$$\sum F_x = 5\cos 30° + 0$$
$$= 4.3N$$
$$\sum F_y = 5\sin 30 - 8$$
$$= \underline{-5.5N}$$



$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{4.3^2 + (-5.5)^2}$$

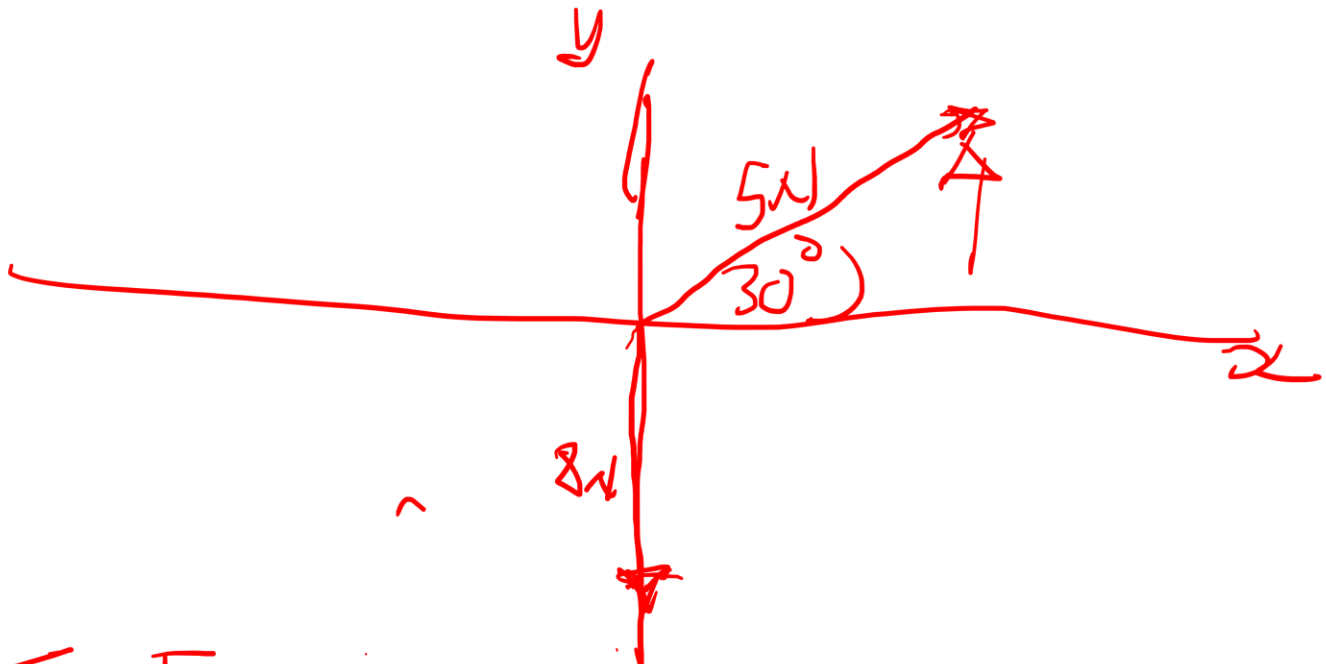
$$F_R = 6.98 N$$

$$\tan\theta = \frac{5.5}{4.3}$$

$$\theta = \tan^{-1}\frac{5.5}{4.3} = 51.98^\circ$$

→ 51.98° below the positive x-axis

→ or $360^\circ - 51.98^\circ =$ ______



$$\sum F_x = 0 + 5\cos 30 = 4.3\,N$$

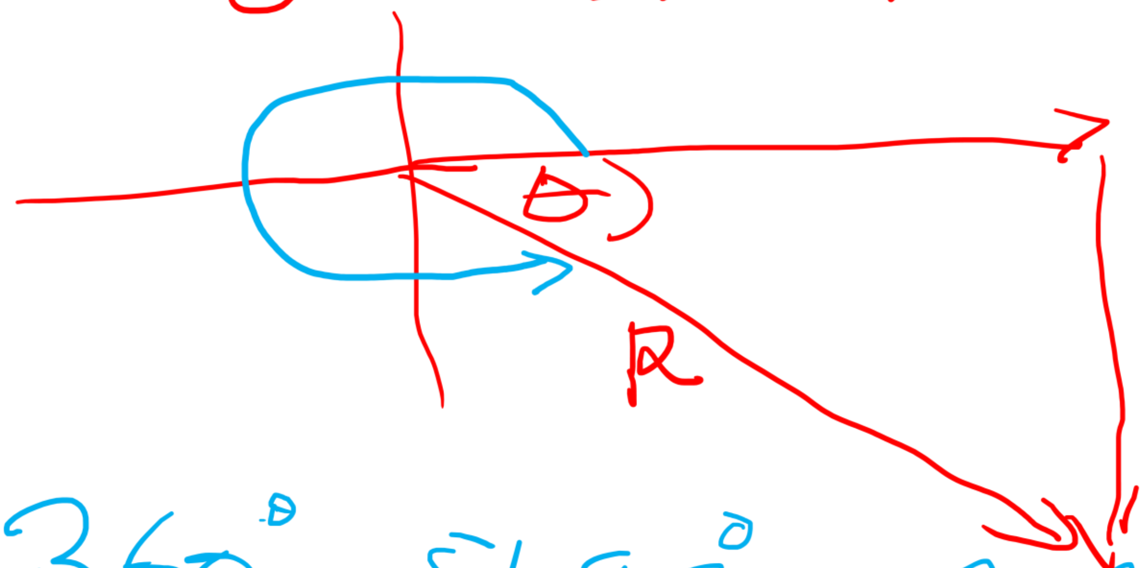
$$\sum F_y = -8 + 5\sin 30 = -5.5\,N$$

$$R = \sqrt{4.3^2 + (-5.5)^2}$$

$$= 6.98\,N$$

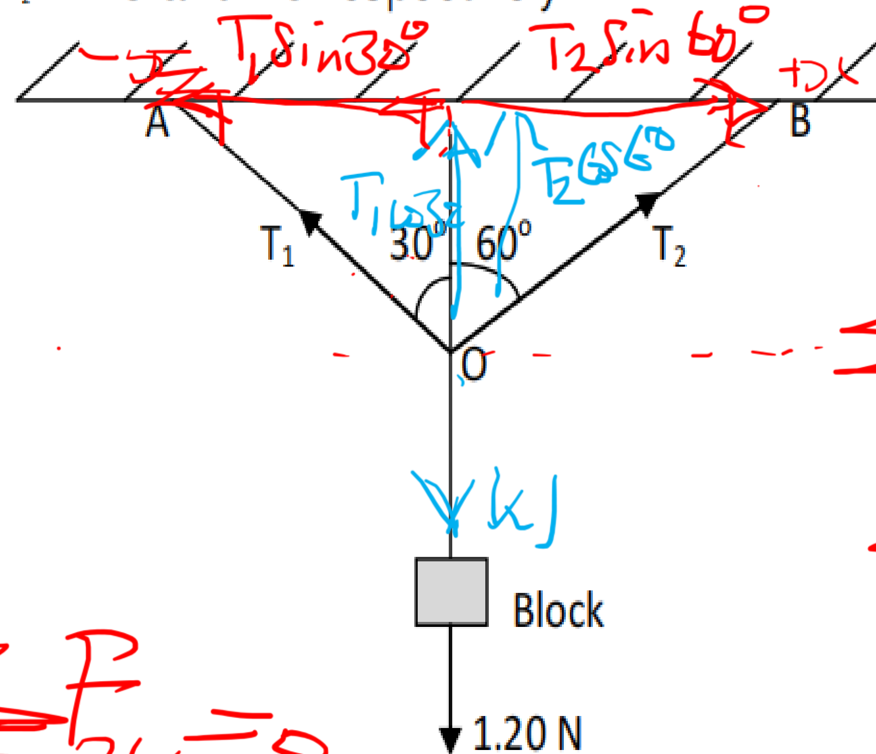
$$\tan\theta = \frac{5.5}{4.3} = 1.2790$$

$$\theta = \tan^{-1} 1.2790 = \underline{51.98^\circ}$$



$$\theta_R = 360^\circ - 51.98^\circ = \underline{\underline{308.02^\circ}}$$

6. A block whose weight is 1.20 N is suspended by a light string which is knotted at O to two other light strings which are attached to the ceiling at A and B. Calculate the tensions T_1 and T_2 in AO and BO respectively.



$\sum F_x = 0$

$\sum F_y = 0$

$\sum F_x = 0$

$-T_1 \sin 30° + T_2 \sin 60° = 0$

$T_1 \sin 30 = T_2 \sin 60$

$T_1 = T_2 \sin 60 / \sin 30$

$$\sum F_y = 0$$

$$T_1 \cos 30 + T_2 \cos 60 + 1 \times 1 = 0$$

$$T_1 \cos 30 + T_2 \cos 60 - 1.2 = 0$$

$$T_1 \cos 30 + T_2 \cos 60 = 1.2$$

$$\frac{T_2 \sin 60}{\sin 30} \times \cos 30 + T_2 \cos 60 = 1.2$$

$$T_2 \left(\frac{\sin 60 \cos 30}{\sin 30} + \cos 60 \right) = 1.2$$

$$T_2 = \frac{1.2}{\frac{\sin 60 \cos 30}{\sin 30} + \cos 60}$$

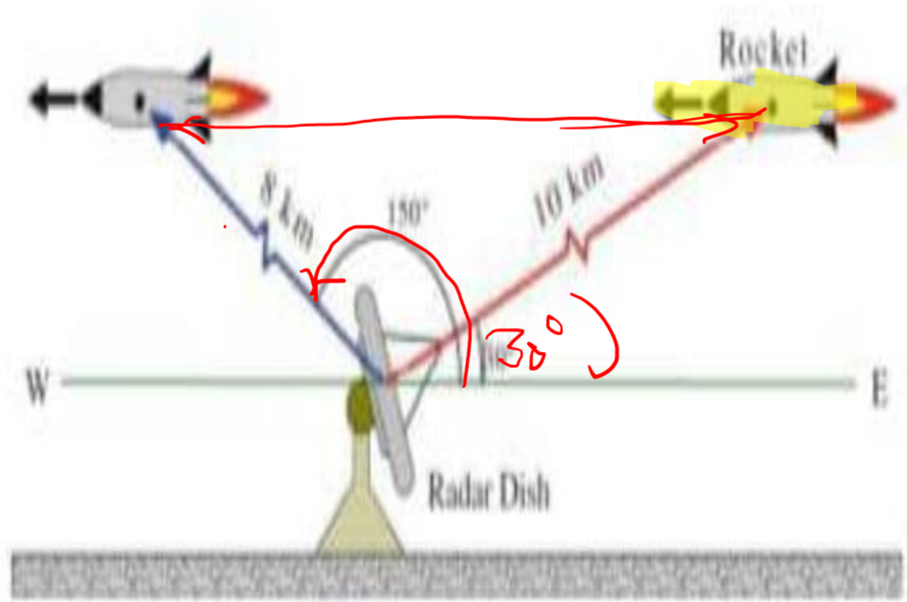
$$\underline{\underline{T_2 = 0.6\,N}}$$

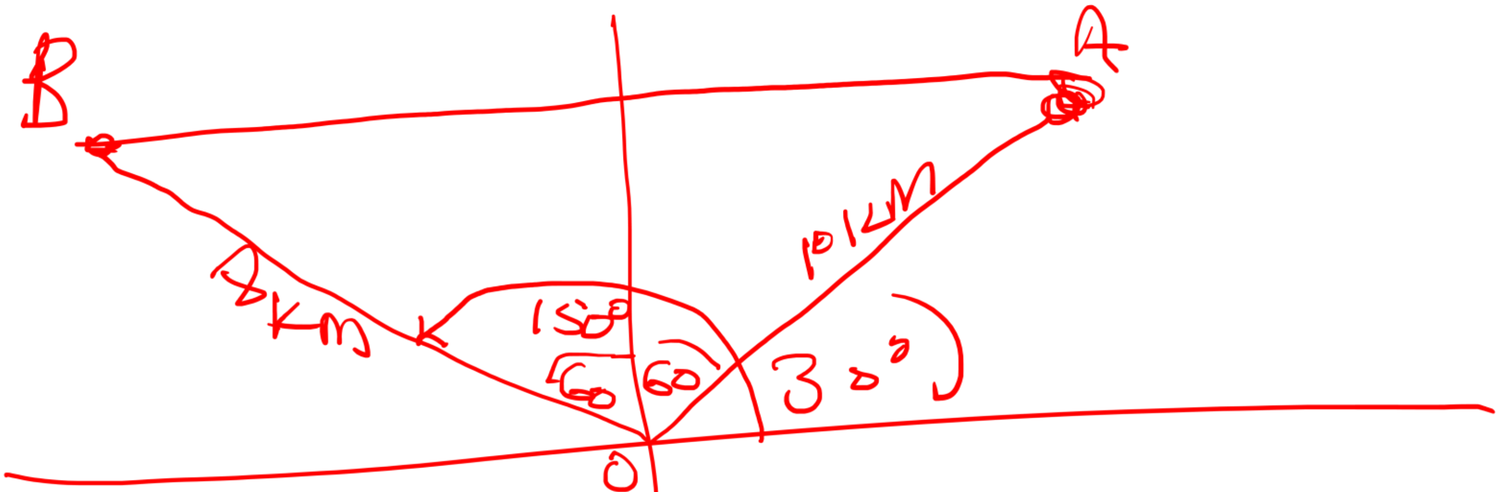
$$T_1 \sin 30 = T_2 \sin 60$$

$$T_1 = \frac{T_2 \sin 60}{\sin 30} = \frac{0.6 \sin 60}{\sin 30}$$

$$\underline{T_1 = 1.039\,N}$$

16. A radar device detects a rocket approaching directly from east due west. At one instant, the rocket was observed 10 km away and making an angle of 30° above the horizon. At another instant the rocket was observed at an angle of 150° in the vertical east-west plane while the rocket was 8 km away, see figure 2.17. Find the displacement of the rocket during the period of observation. [15.62km]





$\boxed{\text{Cosine Rule}}$

$$(AB)^2 = (OA)^2 + (OB)^2 - 2\,OA \times OB \cos 120^\circ$$

$$(AB)^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \cos 120^\circ$$

$$= 100 + 64 - 20 \times 8 \cos 120^\circ$$

$$= 164 - 160 \cos 120^\circ$$

$$AB = \underline{15.62\text{ km}}$$

7. Given the displacement vectors A = (3i – 4j + 4k) m and B = (2i + 3J - 7k) m. Find

(a) components of the resultant displacement and its magnitude

(b) angle between A and B.

(a) $A + B = (3+2)i + (-4+3)j + (4(-7))k$

$A + B = 5i - j - 3k$

$$|A+B| = \sqrt{5^2 + (-1)^2 + (3)^2}$$

$$|A+B| = \sqrt{35}$$

(b) Angle btn A and B

$$A \cdot B = |A||B| \cos\theta$$

$$A \cdot B = 3\times 2 + 3\times -4 + 4\times -7$$

$$A \cdot B = -34$$

$$|A| = \sqrt{3^2 + (-4)^2 + 4^2} = \sqrt{41}$$

$$|B| = \sqrt{2^2 + 3^2 + -7^2} = \sqrt{62}$$

$$A \cdot B = |A|\,|B| \cos\theta$$

$$-34 = \sqrt{41} \times \sqrt{62} \cos\theta$$

$$\cos\theta = \frac{-34}{\sqrt{41} \times \sqrt{62}}$$

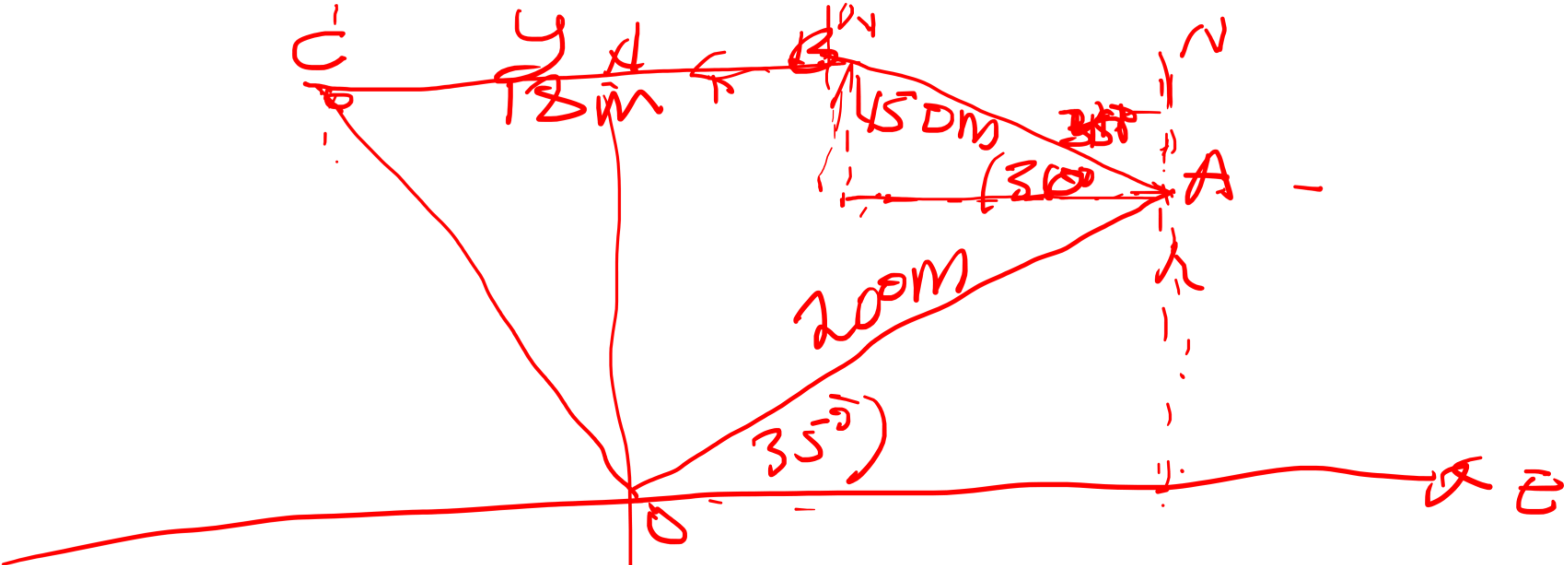
$$\theta = \cos^{-1} \frac{-34}{\sqrt{41} \times \sqrt{62}} = \cos^{-1} -0.674$$

$$\theta = 132.4^{\circ}$$

9. A drone flies from the origin of the coordinate system to point A, located 200 m in the direction 35° north of east. Next, it flies 150 m 30° west of north to point B. Finally it flies 180 m due west to point C. Find the location of point C relative to the origin.

10. A vector is given by **A** = 2**i** + **j** + 3**k**. Find the angles between **A** and the x, y and z axis.

11. A vector is given by **A** = 2**i** + 3**j** + 4**k**, show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Where α, β and γ are angles between **A** and the x, y and z axis respectively.

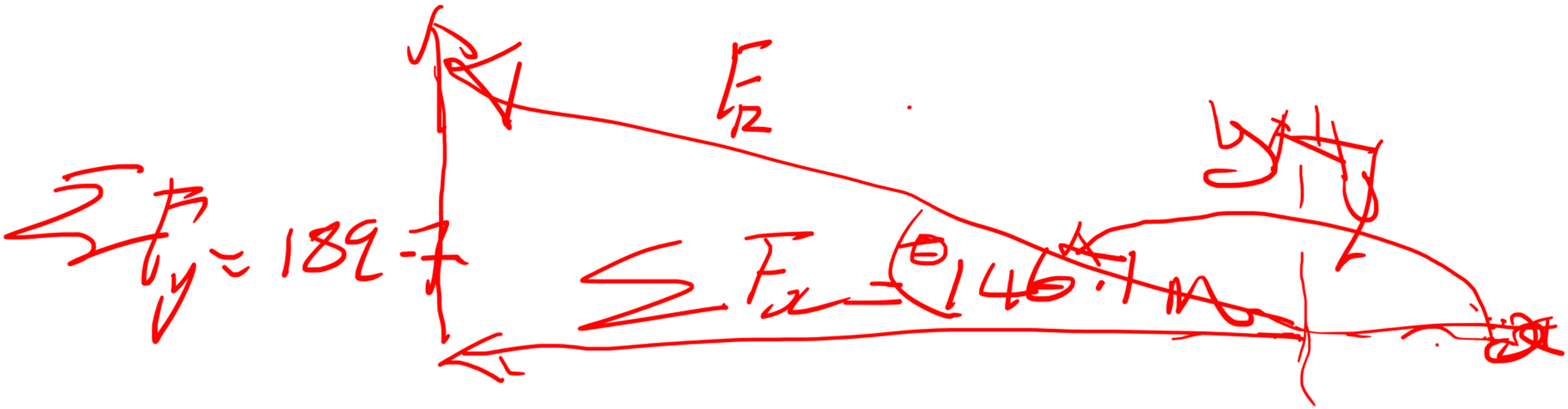


$$\sum F_x = 200\cos 35^\circ - 150\cos 30 - 180$$

$$\sum F_x = -146.1\,m$$

$$\sum F_y = 200\sin 35^\circ + 150\sin 30 + 0$$

$$\sum F_y = 189.7\,m$$



$$F_R^2 = (\Sigma F_x)^2 + (\Sigma F_y)^2$$

$$F_R = \sqrt{(-146.1)^2 + 189.7^2}$$

$$F_R = \sqrt{(-146.1)^2 + 189.7^2}$$

$$F_R = $$

SOH CAH TOA

$$\tan\theta = \frac{189.7}{-146.1}$$

$$\theta = \tan^{-1}\frac{189.7}{-146.1} = \underline{\underline{-52.4^\circ}}$$

$\theta = +52.4^\circ$ above the negative x-axis

$$\text{Actual angle} = 180^\circ - 52.4^\circ = \underline{\underline{127.6^\circ}}$$

4/D. $A = 2i + j + 3K$

$|A| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$

$|A_x| = \sqrt{1^2 + 0^2 + 0^2} = \sqrt{1} = 1$

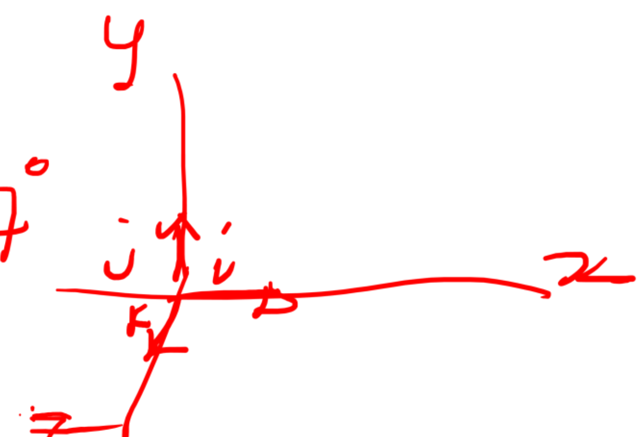
$A \cdot A_x = 2 \times 1 + 1 \times 0 + 3 \times 0 = 2$

$A \cdot A_x = |A|\,|A_x| \cos \alpha$

$2 = \sqrt{14} \times 1 \cos \alpha$

$\cos \alpha = \dfrac{2}{\sqrt{14} \times 1} = \dfrac{2}{\sqrt{14}}$

$\alpha = \cos^{-1} \dfrac{2}{\sqrt{14}} = \underline{57.7^\circ}$



$$|A| = \sqrt{14}$$

$$|A_y| = 0^2 + 1^2 + 0^2 = 1$$

$$A \cdot A_y = |A|\,|A_y| \cos\beta$$

$$= \sqrt{14}$$

$$A \cdot A_y = 2 \times 0 + 1 \times 1 + 3 \times 0 = 1$$

$$1 = \sqrt{14} \times 1 \cos\beta$$

$$\cos\beta = \frac{1}{\sqrt{14}} \Rightarrow \beta = \cos^{-1} \frac{1}{\sqrt{14}} = \underline{74.5^{\circ}}$$

$$|A| = \sqrt{14}$$

$$|A_z| = 0^2 + 0^2 + 1^2 = 1$$

$$A \cdot A_z = 2 \times 0 + 1 \times 0 + 3 \times 1$$

$$A \cdot A_z = 3$$

$$A \cdot A_z = |A| |A_z| \cos \gamma$$

$$3 = \sqrt{14} \times 1 \cos \gamma$$

$$\cos \gamma = \frac{3}{\sqrt{14}} \Rightarrow \gamma = \cos^{-1} \frac{3}{\sqrt{14}}$$

$$\underline{\gamma = 36.7^\circ}$$

411. $A = 2i + 3j + 4k$

Show $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

magnitude of A

$|A| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$

$|A_x| = \sqrt{1^2 + 0^2 + 0^2} = 1$

$|A_y| = \sqrt{0^2 + 1^2 + 0^2} = 1$

$|A_z| = \sqrt{0^2 + 0^2 + 1^2} = 1$

$A \cdot A_x = 2\times1 + 3\times0 + 4\times0 = 2$

$A \cdot A_y = 2\times0 + 3\times1 + 4\times0 = 3$

$A \cdot A_z = 2\times0 + 3\times0 + 4\times1 = 4$

$$A \cdot A_x = |A||A_x| \cos\alpha$$

$$2 = \sqrt{29} \times 1 \cos\alpha$$

$$\cos\alpha = \frac{2}{\sqrt{29}} \quad \text{— (i)}$$

$$A \cdot A_y = |A||A_y| \cos\beta$$

$$3 = \sqrt{29} \times 1 \cos\beta$$

$$\cos\beta = \frac{3}{\sqrt{29}} \quad \text{— (ii)}$$

$$A \cdot A_z = |A||A_z| \cos\gamma$$

$$4 = \sqrt{29} \times 1 \cos\gamma$$

$$\cos\gamma = \frac{4}{\sqrt{29}} \quad \text{— (iii)}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{2}{\sqrt{29}}\right)^2 + \left(\frac{3}{\sqrt{29}}\right)^2 + \left(\frac{4}{\sqrt{29}}\right)^2 = 1$$

$$\frac{4}{29} + \frac{9}{29} + \frac{16}{29} = 1$$

$$\frac{4+9+16}{29} = \frac{29}{29} = 1$$