Dimensional Analysis

- Check mathematical relations for the consistency of their dimensions

Limitations of dimension analysis

- It does not tell whether a given Physical quantity is a scalar or a vector. Size and direction
- It does not always tell us the exact form of a relation
- A dimensionally correct equation may not always be the correct relation. (Because there are more than one physical quantity having the same dimensions)

Dimensional Analysis

Question

For the equation below use dimensional analysis to find the dimension of pressure (**P**) where **F** is force and **A** is area

$$P = \frac{F}{A} \checkmark$$

$$P = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{ma}{A}$$

dimension length [L]

Time = [T]

Mass = [M]

$$P = \frac{F}{A} = \frac{ma}{A}$$

$$= \frac{M \cdot \cancel{L}}{L^{\cancel{2}} \, T^2}$$

$$P = \frac{M}{L \cdot T^2} = \underline{M \cdot L^{-1} \cdot T^{-2}}$$

$a = m/s^2$

$L \times L = Am$

Question

For the equation below, where **E** is kinetic energy, **C** is a constant, **m** is mass and **v** is velocity, use dimension analysis to calculate the values of

$$\propto \text{ and } \beta$$

$$E = Cm^{\alpha}v^{\beta}$$

$$E = C\, m^{\alpha} v^{\beta}$$

Mass $= M$

Velocity $\left(\frac{m}{s}\right) = \frac{L}{T}$

$$KE = \frac{1}{2} m v^2 \Rightarrow \frac{1}{2} M \cdot \frac{L^2}{T^2}$$

$$\frac{1}{2} M \cdot \frac{L^2}{T^2} = C\, M^{\alpha} \frac{L^{\beta}}{T^{\beta}}$$

$$M^1 = M^{\alpha}$$

$$\alpha = 1$$

$$L^2 = L^{\beta}$$

$$\beta = 2$$

$$T^{-2} = T^{-\beta}$$

$$-\beta = -2$$

$$\beta = 2$$

Dimension of work

$W = F \times d$

$= m a \times d$

$\text{mass} = M$

$a = \frac{L}{T^2}$

$d = L$

$W = M \cdot \frac{L}{T^2} \times L$

$W = \frac{M \cdot L^2}{T^2} = M \cdot L^2 \cdot T^{-2}$

Dimension of Power

$$P = \frac{W}{t}$$

$$= \frac{F \times d}{t}$$

$$= \frac{ma \times d}{t}$$

$$= \frac{M \cdot L \times L}{T^2 \times T}$$

$$P = \frac{M \cdot L^2}{T^3}$$

$\text{mass} = M$

$a = \frac{L}{T^2}$

$d = L$

$\text{time} = T$

(i) Write down the dimensions of velocity, acceleration and force. [3]

The force F of gravitational attraction between two objects with masses m_1 and m_2, at a distance r apart, is given by

$$F = \frac{Gm_1m_2}{r^2}$$

where G is the universal constant of gravitation.

(ii) Show that the dimensions of G are $M^{-1}L^3T^{-2}$. [2]

Dimensions of velocity, acceleration and force

(i) Velocity $\left(\frac{m}{s}\right)$

dimension $\frac{L}{T} = LT^{-1}$

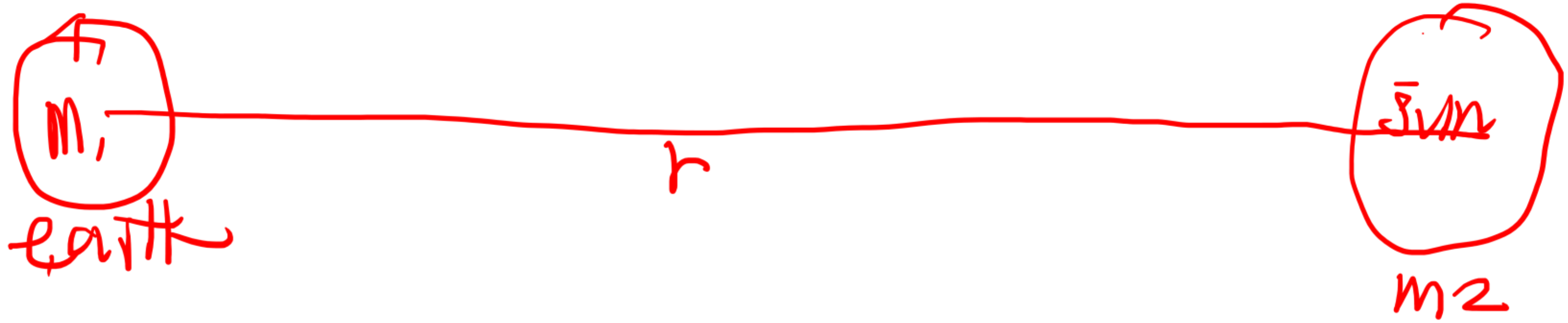
(ii) acceleration (m/s^2)

dimension $\frac{L}{T^2} = LT^{-2}$

(iii) Force

dimension of force (m a)

$$M\frac{L}{T^2} = MLT^{-2}$$



$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

$$F = \frac{G\, m_1\, m_2}{r^2}$$

$$G = \frac{F \times r^2}{m_1 \times m_2}$$

$$= \frac{ma \times r^2}{m_1\, m_2}$$

$$= \frac{M \times \frac{L}{T^2} \times L \times L}{M \times M}$$

$$G = \frac{ML}{T^2} \times L^2 \div M^2$$

$$= \frac{\cancel{M} L \times L^2}{T^2} \times \frac{1}{M^{\cancel{2}}}$$

$$= L^3 T^{-2} M^{-1}$$

$$G = M^{-1} L^3 T^{-2}$$

A moving car experiences a force F due to air resistance. It is known that F depends on a product of powers of its velocity v, its cross-sectional area A and the air density ρ, and is given by

$$F = \frac{1}{2} C \rho^{\alpha} v^{\beta} A^{\gamma},$$

where C is a dimensionless constant known as the drag coefficient.

(i) Write down the dimensions of force and density. [2]

(ii) Use dimensional analysis to find α, β and γ. [5]

(i) Force Dimension $F = ma = M\frac{L}{T^2} = MLT^{-2}$

(ii) Density $\rho = \frac{m}{V} = \frac{M}{L^3} = ML^{-3}$

$$F = \frac{1}{2} C \rho^{\alpha} V^{\beta} A^{\gamma}$$

$$MLT^{-2} = \frac{1}{2} C (ML^{-3})^{\alpha} (LT^{-1})^{\beta} (L^2)^{\gamma}$$

$$MLT^{-2} = \frac{1}{2} C M^{\alpha} L^{-3\alpha} L^{\beta} T^{-\beta} L^{2\gamma}$$

$$M^1 = M^{\alpha}$$

$$\alpha = 1$$

$$T^{-\alpha} = T^{-\beta}$$

$$\frac{-\beta}{-1} = \frac{-\alpha}{-1}$$

$$\beta = 2$$

$$L^1 = L^{-3\alpha} \times L^{\beta} \times L^{2\gamma}$$

$$L^1 = L^{-3\alpha+\beta+2\gamma}$$

$$1 = -3\alpha + \beta + 2\gamma$$

$$1 = -3\times 1 + 2 + 2\gamma$$

$$1 - 2 + 3 = 2\gamma$$

$$\alpha = 2\gamma$$

$$2 = 2\gamma$$

$$\underline{\gamma = 1}$$

In an investigation, small spheres are dropped into a long column of a viscous liquid and their terminal speeds measured. It is thought that the terminal speed V of a sphere depends on a product of powers of its radius r, its weight mg and the viscosity η of the liquid, and is given by

$$V = kr^{\alpha}(mg)^{\beta}\eta^{\gamma},$$

where k is a dimensionless constant.

(i) Given that the dimensions of viscosity are $ML^{-1}T^{-1}$ find α, β and γ. [6]

$$V = K r^{\alpha} (mg)^{\beta} \eta^{\gamma}$$

K is a constant

$$\eta = M L^{-1} T^{-1}$$

$$LT^{-1} = K L^{\alpha} (MLT^{-2})^{\beta} (ML^{-1}T^{-1})^{\gamma}$$

$$LT^{-1} = K L^{\alpha} M^{\beta} L^{\beta} T^{-2\beta} M^{\gamma} L^{-\gamma} T^{-\gamma}$$

$$L^{1} = L^{\alpha} \times L^{\beta} \times L^{-\gamma}$$

$$1 = \alpha + \beta - \gamma \quad \text{——} \quad (i)$$

$$T^{-1} = T^{-2\beta} \times T^{-\gamma}$$

$$-1 = -2\beta - \gamma \quad \text{——} \quad (ii)$$

$$1 = \alpha + \beta - \gamma$$

$$1 = \alpha + 1 - (-1)$$

$$1 = \alpha + 1 + 1$$

$$1 = \alpha + 2$$

$$\alpha = 1 - 2$$

$$\underline{\underline{\alpha = -1}}$$

$$\beta = -\gamma$$

$$\gamma = -1$$

$$\beta = -(-1)$$

$$\underline{\beta = 1}$$